Number Theory Round

20 minutes | 12 problems

1. To securely enter his spaceship, Frank needs to solve a verification problem - after interacting with hundreds of sneaky aliens a day, he needs to make sure there are no imposters on his ship. When he returns today, the screen reads, “Please enter the least common multiple of 12 and 16.” What should Frank enter to prove he is a human?

2. Timothy is in a serious situation where he needs to dock two of his ports onto a rotating satellite. The satellite has 6 separate ports; how many different combinations are possible?

3. Scientists are reviewing photos taken from two satellites. The satellite Vanguard IX takes a photo every 12 minutes, while the satellite Envisat V takes a photo every 20 minutes. Both satellites take their first photo at 12:00 PM. What is the next time they will take a photo at the same time again?

4. The FBI wants to kill a group of 20 aliens standing in a line. To do so, they kill the first alien, then the third, then the fifth, and so on. After they’ve gone through all the aliens, they go back to the start and repeat the same process. After doing this many times, they reach the last alien. What position in line was this alien originally?

5. Jonathan is trying to solve an inscription on the wall left behind by the astronaut previously stationed on Mercury. It reads, “what is the value of $\cfrac{1}{1+\cfrac{5}{1+\cfrac{3}{1+\cfrac{7}{1+9}}}}$?”

6. Grayson builds an AI that classifies 15 planets in a solar system, each at a unique distance from its star. The AI then looks at a group of at least one planet. It marks the group of planets as valid if no two planets in the group share a neighbor (the neighbor does not have to be in the group). How many planets are in the biggest group the AI marks as correct?

7. Lyla is trying to launch her spaceship, but can’t remember the last two digits in her three-digit security code. However, she does remember that the entire code is a number divisible by both 8 and 12, and that the first digit is 2. What is the sum of all possible codes for Lyla’s spaceship?

8. The James Webb Telescope detects 67 planets in 3 separate galaxies, which are Andromeda, Bear Paw, and Centaurus. If the difference in number of planets between Andromeda and Bear Paw is 20, and each galaxy has a prime number of planets, what is the sum of possible planets in the galaxy Centaurus?

9. A solar system has six planets, each a different distance from their star. These planets are either icy, fiery, or habitable, but it is unknown if there exists a planet of every type. Scientists observe that no two consecutive planets are the same type. How many possible solar systems are there?

10. Luke Skywalker has a LOT of parmesan cheese wheels. He can evenly distribute his cheese in 60 distinct ways. What is the least amount of cheese he could have?

11. A galaxy has a large number of planets. When the planets are counted in groups of five, three are left over. When they are counted in groups of six, four are left over. When they are counted in groups of seven, two are left over. How many planets are left over when they are counted in groups of 210?

12. Luke has discovered a wormhole. Going through it, he travels either 5 minutes forward or backward (with equal probability) in time. He then waits 2 minutes, and goes through it again. What is the probability that right after going through the wormhole for the 6th time, he witnesses himself step through the wormhole for the first time?